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Linear Discrete-time Systems with Markovian Jumps and Mode Dependent Time-delay: Stability and Stabilizability

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This paper considers stochastic stability and stochastic stabilizability of linear discrete-time systems with Markovian jumps and mode-dependent time-delays. Linear matrix inequality (LMI) techniques are used to obtain sufficient conditions for the stochastic stability and stochastic stabilizability of this class of systems. A control design algorithm is also provided. A numerical example is given to demonstrate the effectiveness of the obtained theoretical results.

Key words: Discrete-time systems, time-delay, stochastic systems, stability, stabilizability

1 INTRODUCTION

Time-delay is an important factor that may affect the performance of dynamical systems. It can even, in some situation, causes instability of a system that we would like to control if the presence of such time-delay during the design phase is not taken into account. For deterministic linear systems with time-delay, we have seen an increasing interest during the last two decades. There are numerous results in the literature on time-delay systems, see for example, [11] and the references therein. However, most results are on deterministic continuous-time linear systems with time-delay. Stability, stabilizability and control problems for this class of systems have been studied and numerous results are available in the literature such as [5, 6, 8–10, 12, 14, 15]. However, for deterministic class of discrete-time linear systems with time-delay only few results have been reported in the literature. We believe that the main reason for this is that these systems can be transformed to equivalent systems without time-delay and then current results on stability, stabilizability and control design can be applied. For more information on this class of systems, we refer reader to Boukas and Liu [3] and the references therein.

The system we are studying in this paper is referred to as the discrete-time Markovian jump linear systems with time-delay. It has two components in the state vector. The first part

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is called as the state of the system which is continuous-valued, and the second part is regarded as the mode of the system which takes discrete values.

The problems of stochastic stability and stochastic stabilizability have been tackled by many authors and recently we have seen the publication of different results ranging from delay-independent to delay-dependent mainly for the continuous-time case such as [3, 10, 15]. Several situations were considered including constant time-delay, time-varying-delay and mode-dependent time-delay for the continuous-time systems and constant time-delay for discrete-time systems such as [13]. However, to the best of authors' knowledge, the mode-dependent time-delay case for the class of discrete-time linear systems with Markovian jump parameters has not been addressed and this will be the subject of this paper.

The goal of this paper is to develop mode-dependent sufficient conditions for stochastic stability and stochastic stabilizability for linear discrete-time systems with Markovian jump parameters and mode-dependent time-delay in the state. The linear matrix inequality (LMI) techniques described in [4] are used to solve the above problems.

The paper is organized as follows. In Section 2, the problem is stated and the objective of the paper is formulated. The problem of stochastic stability for the given system is examined and delay-dependent sufficient condition is developed in Section 3. We continue in Section 4, to investigate the problem of stabilizability and establish delay-dependent conditions. In addition, a design algorithm that stabilizes the resulting closed-loop system is provided. A numerical example is given in Section 5 to illustrate the proposed theoretical results.

2 PROBLEM STATEMENT

Let $\{r_k, k \geq 0\}$ be a homogeneous Markov chain that takes values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with transition probabilities p_{ij} from mode i to mode j defined by:

$$p_{ij} = \Pr(r_{k+1} = j | r_k = i) \tag{1}$$

where $p_{ij} \geq 0$ and $\sum_{j=1}^{N} p_{ij} = 1$ for all $i, j \in \mathcal{S}$.

The dynamics of the system is assumed to be described by the following set of equations:

$$\sum : \begin{cases} x_{k+1} = A(r_k)x_k + A_d(r_k)x_{k-d_{r_k}} + B(r_k)u_k \\ x_l = \phi_l, \quad l = -d_{r_0}, \ldots, -1, 0. \end{cases} \tag{2}$$

the discrete-time index k is assumed to take values $[-d_{r_k}, \ldots, 0]$ and r_0 is assumed to be given. Here x_k and u_k are the n- and m-dimensional state and control input vectors respectively at instant k. The matrices $A(r_k), A_d(r_k)$ and $B(r_k)$ are constant matrices of appropriate sizes for any fixed values of r_k in $\mathcal{S}$, and d_{r_k} is a positive integer representing the time-delay of the system. Note that since r_k is a Markov chain, so is d_{r_k} with the same transition probabilities. ϕ_k is some initial condition for the state vector x_k. We assume in this paper that d_{r_k} is mode-dependent.

Our objective in this paper is to obtain sufficient conditions that guarantee the stochastic stability and the stochastic stabilizability of the class of system under consideration.

For the system Σ in (2), we use the following control law:

$$u_k = K(r_k)x_k \tag{3}$$

where the control gain $K(r_k)$ is to be designed for each system mode $r_k \in \mathcal{S}$.

Notation The notation used in this paper is quite standard. $\mathbb{Z}$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the set of integer numbers, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$

(respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive-definite). $P = (P(1), \ldots, P(N)) > 0$ means all $P(1), \ldots, P(N)$ are symmetric and positive-definite. I is the identity matrix with compatible dimension. $\mathrm{E}\{\cdot\}$ denotes the expectation operator with respective to certain probability measure P. $\ell_2[0, \infty]$ is the space of square summable vector sequence over $[0, \infty]$. $\|\cdot\|$ will refer to the Euclidean vector norm whereas $\|\cdot\|_{[0,\infty]}$ denotes the $\ell_2[0, \infty]$-norm over $[0, \infty]$ defined as $\|f\|^2_{[0,\infty]} = \sum_0^\infty \|f_k\|^2$. For simplicity of presentation, we define $\underline{d}, \bar{d}, \underline{p}$ as:

$$
\begin{aligned}
\underline{d} &= \min(d_1, \ldots, d_N)\\
\bar{d} &= \max(d_1, \ldots, d_N)\\
\underline{p} &= \min(p_{11}, \ldots, p_{NN})
\end{aligned}
$$

For a given set of positive definite matrices $P(j)$ for $j = 1, \ldots, N$, we define the following convex combinations $\bar{P}(i)$ as

$$
\bar{P}(i) = \sum_{j=1}^{N} p_{r_k j} P(j) \tag{4}
$$

where $i = 1, \ldots, N$.

Remark 1 Notice that, due to the existence of time-delay, d_{r_k}, in the system Σ, the joint state vector (x_k, r_k) is not a Markov chain, however we can define $X_k = (x_{k-d_{r_k}}, x_{k-d_{r_k}+1}, \ldots, x_k)$ to overcome this and get a Markov chain, thus it can be seen that (X_k, r_k) is a Markov chain.

Remark 2 Indeed, system Σ in (2) can be reformulated as a one without time-delay by constructing a higher dimensional vector $X(k)$ defined as:

$$
X(k) = \begin{bmatrix} x(k) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-\underline{d}) \end{bmatrix} \tag{5}
$$

Then, system Σ can be rewritten as

$$
\begin{aligned}
X(k+1) &= F(r_k)X(k) + G(r_k)u(k) &(6)\\
x(k) &= HX(k) &(7)
\end{aligned}
$$

where

$$
F(r_k) = \begin{bmatrix}
A(r_k) & 0 & \cdots & A_d(r_k) & 0 & \cdots & 0\\
I & 0 & \cdots & 0 & \cdots & \cdots & 0\\
0 & I & \cdots & 0 & \cdots & \cdots & 0\\
\vdots & 0 & \ddots & & & & \vdots\\
\vdots & & & \ddots & & & \vdots\\
\vdots & \vdots & & & \ddots & & \\
0 & 0 & 0 & \cdots & 0 & I & 0
\end{bmatrix}, \tag{8}
$$

$$G(r_k) = \begin{bmatrix} B(r_k) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad H = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix}. \tag{9}$$

Note that system (6) and (7) is time-delay free, for which, the issues of stochastic stability, stabilization and control can be sorted out by using standard existing results on linear discrete-time Markovian jump systems, for example, Ji *et al.* [7]. In this paper, instead of using the extending system dimension approach, we propose a different methodology to handle system (2), by which the maximum unknown time-delay can be conventionally determined such that the underlying system remains stochastically stable. Notice that this point is very important since it is difficult to know time-delay exactly in practice, thus using upper and lower bounds in design would be an alternative effective way.

In next section, we consider the stability of the system Σ given in (2).

3 STABILITY

In this section, we present delay-dependent conditions that can be used to check whether the system Σ in (2) is stochastically stable or not. First, we introduce the following stochastic stability concept.

DEFINITION 1 *For the system Σ in* (2) *with $u_k = 0$ for all $k \geq 0$, the equilibrium point* 0 *is stochastically stable, if for every initial state, (x_0, r_0), the following holds*:

$$\mathrm{E}\left\{ \sum_{k=0}^{\infty} \|x_k(\cdot)\|^2 | x_0, r_0 \right\} < \infty. \tag{10}$$

In the rest of this paper, we will use x_k to denote the solution, $x_k(x_0, r_0)$ when the initial condition, (x_0, r_0) are given. We will also use $X_k = \{x_{k-d_{r_k}}, x_{k-d_k+1}, \ldots, x_k\}$.

The following result concerns with stochastic stability of the system Σ in (2).

THEOREM 1 *If there exist symmetric and positive-definite matrices $P = (P(1), \ldots, P(N)) > 0$ and $Q > 0$ such that the following holds for every $r_k \in \mathcal{S}$*:

$$M(r_k) = \begin{bmatrix} M_{11}(r_k) & A^\top(r_k)\bar{P}(r_k)A_d(r_k) \\ A_d^\top(r_k)\bar{P}(r_k)A(r_k) & A_d^\top(r_k)\bar{P}(r_k)A_d(r_k) - Q \end{bmatrix} < 0 \tag{11}$$

with $M_{11}(r_k) = A^\top(r_k)\bar{P}(r_k)A(r_k) - P(r_k) + (1 + (\bar{d} - \underline{d})(1 - \underline{p}))Q$, then the system Σ in (2) *is stochastically stable.*

Proof To prove our theorem, let us consider the following Lyapunov candidate functional:

$$V(X_k, k) = V_1(X_k, k) + V_2(X_k, k) + V_3(X_k, k)$$